SADLER MATHEMATICS METHODS UNIT 3

WORKED SOLUTIONS

Chapter 3 Antidifferentiation

Exercise 3A

Question 1

$$\frac{dy}{dx} = x^6$$

$$y = \frac{x^7}{7} + c$$

Question 2

$$\frac{dy}{dx} = x^3$$

$$y = \frac{x^4}{4} + c$$

Question 3

$$\frac{dy}{dx} = 10x^4$$

$$y = \frac{10x^5}{5} + c$$

$$= 2x^5 + c$$

Question 4

$$\frac{dy}{dx} = 7x^2$$

$$y = \frac{7x^3}{3} + c$$

Question 5

$$\frac{dy}{dx} = 8x$$
$$y = \frac{8x^2}{2} + c$$
$$= 4x^2 + c$$

Question 6

$$\frac{dy}{dx} = 8$$
$$y = 8x + c$$

Question 7

$$\frac{dy}{dx} = x^{\frac{1}{2}}$$
$$y = \frac{2}{3}x^{\frac{3}{2}} + c$$

Question 8

$$\frac{dy}{dx} = x^{\frac{1}{3}}$$
$$y = \frac{3}{4}x^{\frac{4}{3}} + c$$

Question 9

$$\frac{dy}{dx} = x^{\frac{5}{2}}$$
$$y = \frac{2}{7}x^{\frac{7}{2}} + c$$

Question 10

$$\frac{dy}{dx} = 6x^{\frac{3}{2}}$$
$$y = 6 \times \frac{2}{5}x^{\frac{5}{2}} + c$$
$$= \frac{12}{5}x^{\frac{5}{2}} + c$$

Question 11

$$\frac{dy}{dx} = 4x^{-\frac{1}{2}}$$

$$y = 4 \times 2x^{\frac{1}{2}} + c$$

$$= 8x^{\frac{1}{2}} + c$$

Question 12

$$\frac{dy}{dx} = 4x^{-\frac{1}{2}}$$

$$y = 8\sqrt{x} + c$$

Question 13

$$\frac{dy}{dx} = 10x^{-4}$$

$$y = \frac{10x^{-3}}{-3} + c$$

$$= -\frac{10}{3x^3} + c$$

Question 14

$$\frac{dy}{dx} = -9x^{-2}$$

$$y = \frac{-9x^{-1}}{-1} + c$$

$$= \frac{9}{x} + c$$

Question 15

$$\frac{dy}{dx} = -16x^{-\frac{1}{2}}$$

$$y = -16x^{\frac{1}{2}}.2 + c$$

$$= -32\sqrt{x} + c$$

Question 16

$$\frac{dy}{dx} = 6x^2 - 4x + 3$$
$$y = 2x^3 - 2x^2 + 3x + c$$

Question 17

$$\frac{dy}{dx} = 12x^2 + 3$$
$$y = 3x^4 + 3x + c$$

Question 18

$$\frac{dy}{dx} = x^3 + 3x^2 + 2x$$
$$y = \frac{x^4}{4} + x^3 + x^2 + c$$

Question 19

$$\frac{dy}{dx} = 1 + 4x + 18x^2$$
$$y = x + 2x^2 + 6x^3 + c$$

Question 20

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + 6x$$
$$y = 3x^{\frac{3}{2}} \times \frac{2}{3} + 3x^2 + c$$
$$= 2x^{\frac{3}{2}} + 3x^2 + c$$

Question 21

$$\frac{dy}{dx} = 3x^2 + 14x + 8$$
$$y = x^3 + 7x^2 + 8x + c$$

Question 22

$(3x+2)(x+4) = 3x^2 + 14x + 8$

$\frac{dy}{dx} = 3x^2 + 14x + 8$

$y = x^3 + 7x^2 + 8x + c$

Question 23

$\frac{dy}{dx} = x^2 + 4x - 12$

$y = \frac{x^3}{3} + 2x^2 - 12x + c$

Question 24

$\frac{dy}{dx} = 9x^2 - 4$

$y = 3x^3 - 4x + c$

Question 25

$\frac{dy}{dx} = 12x^3 + 12x$

$y = 3x^4 + 6x^2 + c$

Question 26

$\frac{dy}{dx} = 4x + 5$

$y = 2x^2 + 5x + c$

Question 27

$\frac{dy}{dx} = 2x^{-2} + x^{-3}$

$y = -2x^{-1} + \frac{x^{-2}}{-2} + c$

$= -\frac{2}{x} - \frac{1}{2x^2} + c$

Question 28

$$\frac{dy}{dx} = 6x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$$

$$y = 6 \times \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} \times 2 + c$$

$$= 4x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + c$$

Question 29

$$\frac{dy}{dx} = x^{-\frac{1}{2}} - x^{\frac{3}{2}}$$

$$y = 2x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$$

$$= 2\sqrt{x} - \frac{2}{5}\sqrt{x^5} + c$$

Question 30

$$\frac{dy}{dx} = x^{-\frac{1}{2}} + 1$$

$$y = 2x^{\frac{1}{2}} + x + c$$

$$= 2\sqrt{x} + x + c$$

Question 31

$$\frac{dy}{dx} = 6x^2 + 1$$

$$y = 2x^3 + x + c$$

$$13 = 2(8) + 2 + c$$

$$c = -5$$

$$\therefore y = 2x^3 + x - 5$$

Question 32

$$\frac{dy}{dx} = 4x - 3$$
$$y = 2x^2 - 3x + c$$
$$29 = 2(-3)^2 - 3(-3) + c$$
$$29 = 27 + c$$
$$c = 2$$
$$\therefore y = 2x^2 - 3x + 2$$

Question 33

$$\frac{dA}{dt} = 1 - 6t^{-2}$$
$$A = t + 6t^{-1} + c$$
$$-2 = 2 + 3 + c$$
$$c = -7$$
$$\therefore A = t + \frac{6}{t} - 7$$

Question 34

$$\frac{dv}{dx} = x + x^{-\frac{1}{2}}$$
$$v = \frac{x^2}{2} + 2x^{\frac{1}{2}} + c$$
$$2 = 8 + 4 + c$$
$$c = -10$$
$$\therefore v = \frac{x^2}{2} + 2\sqrt{x} - 10$$

Question 35

a $f'(x) = \frac{6x^2}{5} - \frac{5}{6}x^{-2}$

$$f(x) = \frac{6}{5} \times \frac{x^3}{3} + \frac{5}{6}x^{-1}$$
$$= \frac{2x^3}{5} + \frac{5}{6x} + c$$

$$51 = \frac{2}{5} \times 125 + \frac{5}{6(5)} + c$$
$$c = 51 - 50 - \frac{1}{6}$$
$$= \frac{5}{6}$$
$$f(x) = \frac{2x^3}{5} + \frac{5}{6x} + \frac{5}{6}$$

b $f(1) = \frac{2}{5} + \frac{5}{6} + \frac{5}{6}$

$$= \frac{31}{15}$$

c $f(-1) = -\frac{2}{5} - \frac{5}{6} + \frac{5}{6}$

$$= -\frac{2}{5}$$

Exercise 3B

Question 1

$$\int (3x+2)^3 dx$$
$$= \frac{1}{3}\int 3(3x+2)^3 dx$$
$$= \frac{1}{3} \times \frac{(3x+2)^4}{4} + c$$
$$= \frac{1}{12}(3x+2)^4 + c$$

Question 2

$$\int (3x+2)^4 dx$$
$$= \frac{1}{3}\int 3(3x+2)^4 dx$$
$$= \frac{1}{3} \times \frac{(3x+2)^5}{5} + c$$
$$= \frac{1}{15}(3x+2)^5 + c$$

Question 3

$$\int x(3x+2) dx$$
$$= \int (3x^2 + 2x)\, dx$$
$$= x^3 + x^2 + c$$

Question 4

$$\int (1+5x)^4 dx$$
$$= \frac{1}{5}\int 5\,(1+5x)^4\, dx$$
$$= \frac{1}{5} \times \frac{(1+5x)^5}{5} + c$$
$$= \frac{1}{25}(1+5x)^5 + c$$

Question 5

$$\int (1-5x)^3 dx$$
$$= -\frac{1}{5}\int (-5)\,(1-5x)^3 dx$$
$$= -\frac{1}{5} \times \frac{(1-5x)^4}{4} + c$$
$$= -\frac{1}{20}(1-5x)^4 + c$$

Question 6

$$\int 10x(x^2+5)^4 dx$$
$$= 5\int 2x(x^2+5)^4 dx$$
$$= 5 \times \frac{(x^2+5)^5}{5} + c$$
$$= (x^2+5)^5 + c$$

Question 7

$$\int 20x(x^2-7)^4 dx$$
$$= 10\int 2x(x^2-7)^4 dx$$
$$= 10 \times \frac{(x^2-7)^5}{5} + c$$
$$= 2(x^2-7)^5 + c$$

Question 8

$$x(1+5x)^2$$
$$= x(1+10x+25x^2)$$
$$= x+10x^2+25x^3$$
$$\int x+10x^2+25x^3 dx$$
$$= \frac{x^2}{2} + \frac{10x^3}{3} + \frac{25x^4}{4} + c$$

Question 9

$$\int (2x+1)^2 dx$$
$$= \frac{1}{2}\int 2(2x+1)^2 dx$$
$$= \frac{1}{2} \times \frac{(2x+1)^3}{3} + c$$
$$= \frac{1}{6}(2x+1)^3 + c$$

Question 10

$$x(2x+1)^2$$
$$= x(4x^2 + 4x + 1)$$
$$= 4x^3 + 4x^2 + x$$
$$\int (4x^3 + 4x^2 + x)\, dx$$
$$= x^4 + \frac{4x^3}{3} + \frac{x^2}{2} + c$$

Question 11

$$\int (5x+1)^3 dx$$
$$= \frac{1}{5}\int 5(5x+1)^3 dx$$
$$= \frac{1}{5} \times \frac{(5x+1)^4}{4} + c$$
$$= \frac{1}{20}(5x+1)^4 + c$$

Question 12

$$\int 21(5-7x)^3 dx$$
$$= -3\int (-7)\,(5-7x)^3 dx$$
$$= -3 \times \frac{(5-7x)^4}{4} + c$$
$$= -\frac{3(5-7x)^4}{4} + c$$

Question 13

$$\int 16(2x+1)^3 dx$$
$$= 8\int 2(2x+1)^3 dx$$
$$= 8 \times \frac{(2x+1)^4}{4} + c$$
$$= 2(2x+1)^4 + c$$

Question 14

$$\int 45(3x-2)^4 dx$$
$$= 15\int 3(3x-2)^4 dx$$
$$= 15 \times \frac{(3x-2)^5}{5} + c$$
$$= 3(3x-2)^5 + c$$

Question 15

$$\frac{d}{dx}(x^2 - x + 3) = 2x - 1$$

$$\int (x^2 - x + 3)^4 (2x-1)\, dx$$
$$= \frac{(x^2 - x + 3)^5}{5} + c$$

Question 16

$$\int 48(6x+1)^3 dx$$
$$= 8\int 6(6x+1)^3 dx$$
$$= 8 \times \frac{(6x+1)^4}{4} + c$$
$$= 2(6x+1)^4 + c$$

Question 17

$$\int 2(5x+1)^3 dx$$
$$= \frac{2}{5}\int 5(5x+1)^3 dx$$
$$= \frac{2}{5} \times \frac{(5x+1)^4}{4} + c$$
$$= \frac{1}{10}(5x+1)^4 + c$$

Question 18

$$\frac{d}{dx}(3x^2 - 6x + 1) = 6x - 6$$
$$= 6(x-1)$$
$$\int 150(x-1)(3x^2 - 6x + 1)\, dx$$
$$= 25\int 6(x-1)(3x^2 - 6x + 1)^4 dx + c$$
$$= 25 \times \frac{(3x^2 - 6x + 1)^5}{5} + c$$
$$= 5(3x^2 - 6x + 1)^5 + c$$

Question 19

$$\int 5(3x-1)^4 dx$$
$$= \frac{5}{3}\int 3(3x-1)^4 dx$$
$$= \frac{5}{3} \times \frac{(3x-1)^5}{5} + c$$
$$= \frac{1}{3}(3x-1)^5 + c$$

Question 20

$$\int 3(9x+1)^2 dx$$
$$= \frac{1}{3}\int 9(9x+1)^2 dx$$
$$= \frac{1}{3} \times \frac{(9x+1)^3}{3} + c$$
$$= \frac{1}{9}(9x+1)^3 + c$$

Question 21

$$\int x(3x+4)\,dx$$
$$= \int (3x^2+4)x dx$$
$$= x^3 + 2x^2 + c$$

Question 22

$$\int 2(3x-1)^2 dx$$
$$= \frac{2}{3}\int 3(3x-1)^2\,dx$$
$$= \frac{2}{3} \times \frac{(3x-1)^3}{3} + c$$
$$= \frac{2}{9}(3x-1)^3 + c$$

Question 23

$$2x(x-1)^2$$
$$= 2x(x^2 - 2x + 1)$$
$$= 2x^3 - 4x^2 + 2x$$
$$\int 2x(x-1)^2 dx$$
$$= \int (2x^3 - 4x^2 + 2x)dx$$
$$= \frac{2x^4}{4} - \frac{4x^3}{3} + x^2 + c$$
$$= \frac{1}{2}x^4 - \frac{4}{3}x^3 + x^2 + c$$

Question 24

$$(x+1)(x-1) = x^2 - 1$$
$$\int (x^2 - 1)\,dx$$
$$= \frac{1}{3}x^3 - x + c$$

Question 25

$$\int (1+x)^3 dx$$
$$= \frac{1}{4}(1+x)^4 + c$$

Question 26

$$\int (1-x)^3 dx$$
$$= -\int (-1)(1-x)^3 dx$$
$$= -\frac{(1-x)^4}{4} + c$$

Question 27

$$\int x(1+x)\, dx$$
$$\int (x+x^2)\, dx$$
$$= \frac{1}{2}x + \frac{1}{3}x^3 + c$$

Question 28

$$\int 2x(1+x)^2 dx$$
$$\int (2x^3 + 4x^2 + 2x)\, dx$$
$$= \frac{x^4}{2} + \frac{4x^3}{3} + x^2 + c$$

Question 29

$$\int 12x(1+x^2)^2 dx$$
$$= 6\int 2x(1+x^2)^2\, dx$$
$$= 6 \times \frac{(1+x^2)^3}{3} + c$$
$$= 2(1+x^2)^3 + c$$

Question 30

$$\int 2x(1+x^2)^6 dx$$
$$= \frac{(1+x^2)^7}{7} + c$$

Question 31

$$\int -24(1-2x)^3 dx$$
$$= 12\int (-2)(1-2x)^3 dx$$
$$= 12 \times \frac{(1-2x)^4}{4} + c$$
$$= 3(1-2x)^4 + c$$

Question 32

$$\int 54(2x-1)^8 dx$$
$$= 27\int 2(2x-1)^8 dx$$
$$= 27 \times \frac{(2x-1)^9}{9} + c$$
$$= 3(2x-1)^9 + c$$

Question 33

$$\int 15(5-6x)^4 dx$$
$$= -\frac{5}{2}\int (-6)(5-6x)^4 dx$$
$$= -\frac{5}{2} \times \frac{(5-6x)^5}{5} + c$$
$$= -\frac{1}{2}(5-6x)^5 + c$$

Question 34

$$\int (3-2x)^3 dx$$
$$= -\frac{1}{2}\int (-2)(3-2x)^3 dx$$
$$= -\frac{1}{2} \times \frac{(3-2x)^4}{4} + c$$
$$= -\frac{1}{8}(3-2x)^4 + c$$

Question 35

$$\int 6(2x-3)^8 dx$$
$$= 3\int 2(2x-3)^8 dx$$
$$= 3 \times \frac{(2x-3)^9}{9} + c$$
$$= \frac{1}{3}(2x-3)^9 + c$$

Question 36

$$\int 12(5-6x)^3 dx$$
$$= -2\int (-6)(5-6x)^3 dx$$
$$= -2 \times \frac{(5-6x)^4}{4} + c$$
$$= -\frac{1}{2}(5-6x)^4 + c$$

Question 37

$$\frac{d}{dx}(x^2+x+3) = 2x+1$$
$$\int (2x+1)(x^2+x+3)^4 dx$$
$$= \frac{(x^2+x+3)^5}{5} + c$$

Question 38

$$\int 20x(5x^2+3)^7 dx$$
$$= 2\int 10x(5x^2+3)^7 dx$$
$$= 2 \times \frac{(5x^2+3)^8}{8} + c$$
$$= \frac{(5x^2+3)^8}{4} + c$$

Question 39

$$\frac{d}{dx}(x^2 - x + 3) = 2x - 1$$
$$= -1(1 - 2x)$$
$$\int (1 - 2x)(x^2 - x + 3)^4 dx$$
$$= -\int (2x - 1)(x^2 - x + 3)^4 dx$$
$$= -\int (2x - 1)(x^2 - x + 3)^4$$
$$= -\frac{(x^2 - x + 3)^5}{5} + c$$

Question 40

$$\int (x + 2)^{-4} dx$$
$$= \frac{(x + 2)^{-3}}{-3} + c$$
$$= -\frac{1}{3(x + 2)^3} + c$$

Question 41

$$\int 5(x + 1)^{-2} dx$$
$$= 5\int \frac{(x + 1)^{-2}}{-2} dx$$
$$= 5 \times \frac{(x + 1)^{-1}}{-1} + c$$
$$= -\frac{5}{(x + 1)} + c$$

Question 42

$$\frac{d}{dx}(x^2-2x+1)=2x-2$$

$$1-x=-\frac{1}{2}(2x-2)$$

$$\int(1-x)(x^2-2x+1)^3\,dx$$

$$=-\frac{1}{2}\int(2x-2)(x^2-2x+1)^3\,dx$$

$$=-\frac{1}{2}\times\frac{(x^2-2x+1)^4}{4}$$

$$=-\frac{(x^2-2x+1)^4}{8}+c$$

Question 43

$$\int 2(x+3)^{-3}dx$$

$$=2\times\frac{(x+3)^{-2}}{-2}+c$$

$$=-\frac{1}{(x+3)^2}+c$$

Question 44

$$\int 18x(x^2-3)^{-4}dx$$

$$=9\int 2x(x^2-3)^{-4}\,dx$$

$$=9\times\frac{(x^2-3)^{-3}}{-3}+c$$

$$=-\frac{3}{(x^2-3)^3}+c$$

Question 45

$$\int(x-2)^{-2}dx$$

$$=\frac{(x-2)^{-1}}{(-1)}+c$$

$$=-\frac{1}{(x-2)}+c$$

$$=\frac{1}{2-x}+c$$

Question 46

$$\int (2x-1)^{-2}dx$$
$$=\frac{1}{2}\int 2(2x-1)^{-2}dx$$
$$=\frac{1}{2}\times\frac{(2x-1)^{-1}}{-1}+c$$
$$=-\frac{1}{2}\times\frac{1}{(2x-1)}+c$$
$$=\frac{1}{2(1-2x)}+c$$

Question 47

$$\int 20(3-2x)^{-3}dx$$
$$=-10\int(-2)(3-2x)^{-3}dx$$
$$=-10\times\frac{(3-2x)^{-2}}{-2}+c$$
$$=\frac{5}{(3-2x)^2}+c$$

Question 48

$$\frac{d}{dx}(3x^2-x+1)=6x-1$$
$$\int 10(6x-1)(3x^2-x+1)^4dx$$
$$10\int(6x-1)(3x^2-x+1)^4dx$$
$$=10\times\frac{(3x^2-x+1)^5}{5}+c$$
$$=2(3x^2-x+1)^5+c$$

Question 49

$$-\int(x-2)^{-3}dx$$
$$=-\frac{(x-2)^{-2}}{-2}+c$$
$$=\frac{1}{2(x-2)^2}+c$$

Question 50

$$\int 12(3x-1)^{-2}dx$$
$$= 4\int 3(3x-1)^{-2}dx$$
$$= 4\times\frac{(3x-1)^{-1}}{-1}+c$$
$$= \frac{-4}{(3x-1)}+c$$
$$= \frac{4}{1-3x}+c$$

Question 51

$$\int 20(1-5x)^{-3}dx$$
$$= -4\int(-5)(1-5x)^{-3}dx$$
$$= -4\times\frac{(1-5x)^{-2}}{-2}+c$$
$$= \frac{2}{(1-5x)^2}+c$$

Question 52

$$\int (3x+2)^{\frac{1}{2}}dx$$
$$= \frac{1}{3}\int 3(3x+2)^{\frac{1}{2}}dx$$
$$= \frac{1}{3}\times\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}}+c$$
$$= \frac{2}{9}(3x+2)^{\frac{3}{2}}+c$$

Question 53

$$\int 12(2x-5)^{\frac{1}{2}}dx$$
$$= 6\int 2(2x-5)^{\frac{1}{2}}dx$$
$$= 6\times\frac{(2x-5)^{\frac{3}{2}}}{\frac{3}{2}}+c$$
$$= 4(2x-5)^{\frac{3}{2}}+c$$

Question 54

$$\int 6(1+2x)^{-\frac{1}{2}}dx$$
$$= 3\int\ 2(1+2x)^{-\frac{1}{2}}dx$$
$$= 3\times\frac{(1+2x)^{\frac{1}{2}}}{\frac{1}{2}}+c$$
$$= 6\sqrt{1+2x}+c$$

Question 55

$$\int\left(1+(1-5x)^2\right)dx$$
$$= \int 1dx-\frac{1}{5}\int(-5)(1-5x)^2dx$$
$$= x-\frac{1}{5}\frac{(1-5x)^3}{3}+c$$
$$= x-\frac{1}{15}(1-5x)^3+c$$

Question 56

$$\int 12(3x-2)^{\frac{1}{3}}dx$$
$$= 4\int 3(3x-2)^{\frac{1}{3}}dx$$
$$= 4\times\frac{(3x-2)^{\frac{4}{3}}}{\frac{4}{3}}+c$$
$$= 3(3x-2)^{\frac{4}{3}}+c$$

Question 57

$$\int 1+x(1-5x)^2dx$$
$$= \int(1+x-10x^2+25x^3)\,dx$$
$$= x+\frac{1}{2}x^2-\frac{10x^3}{3}+\frac{25x^4}{4}+c$$

Question 58

$$\int 12(2x-3)^{-4}dx$$
$$= 6\int\ 2(2x-3)^{-4}\,dx$$
$$= 6\times\frac{(2x-3)^{-3}}{-3}+c$$
$$= -\frac{2}{(2x-3)^3}+c$$

Question 59

$$\int\left(12(2x+1)^2+9(3x-2)^2\right)dx$$
$$= 6\int 2(2x+1)^2\,dx+3\int 3(3x-2)^2\,dx$$
$$= 6\times\frac{(2x+1)^3}{3}+3\times\frac{(3x-2)^3}{3}+c$$
$$= 2(2x+1)^3+(3x-2)^3+c$$

Question 60

$$\int\left((x+3)^{\frac{1}{2}}+(x+1)^{\frac{1}{2}}\right)dx$$
$$= \frac{2}{3}(x+3)^{\frac{3}{2}}+\frac{2}{3}(x+1)^{\frac{3}{2}}+c$$

Question 61

$$\frac{d}{dx}(x^2+3x-1)=2x+3$$
$$10x+15=5(2x+3)$$
$$\int(10x+15)(x^2+3x-1)^{-\frac{1}{2}}dx$$
$$= 5\int(2x+3)(x^2+3x-1)^{-\frac{1}{2}}dx$$
$$= 5\times\frac{(x^2+3x-1)^{\frac{1}{2}}}{\frac{1}{2}}+c$$
$$= 10(x^2+3x-1)^{\frac{1}{2}}+c$$
$$= 10\sqrt{x^2+3x-1}+c$$

Question 62

$$\frac{dA}{dp} = 6(p+1)^2$$

$$A = 6\int (p+1)^2 dx$$

$$= \frac{6(p+1)^3}{3} + c$$

$$= 2(p+1)^3 + c$$

$$21 = 16 + c$$

$$c = 5$$

$$\therefore A = 2(p+1)^3 + 5$$

Question 63

$$y = \int 20(2x+1)^4 \, dx$$

$$= 10\int 2(2x+1)^4 \, dx$$

$$= 10 \times \frac{(2x+1)^5}{5} + c$$

$$= 2(2x+1)^5 + c$$

$$25 = 2 + c$$

$$c = 23$$

$$\therefore y = 2(2x+1)^5 + 23$$

Question 64

$$f'(x) = 32(3-2x)^3$$

$$f(x) = -16\int (-2)(3-2x)^3 dx$$

$$= -16 \times \frac{(3-2x)^4}{4} + c$$

$$= -4(3-2x)^4 + c$$

$$1 = -4 + c$$

$$c = 5$$

$$\therefore f(x) = -4(3-2x)^4 + 5$$

Question 65

$$\frac{dy}{dx} = 15x(5x^2-1)^2 = \frac{3}{2} \times 10x(5x^2-1)^2$$

$$y = \frac{3}{2}\int 10x(5x^2-1)^2 dx$$

$$= \frac{3}{2} \times \frac{(5x^2-1)^3}{3} + c$$

$$= \frac{(5x^2-1)^3}{2} + c$$

$$40 = 32 + c$$

$$c = 8$$

$$\therefore y = \frac{(5x^2-1)^3}{2} + 8$$

Question 66

$$v = \int 100t(t^2+1)^{-3} dx$$

$$= 50\int 2t(t^2+1)^{-3} dx$$

$$= 50 \times \frac{(t^2+1)^{-2}}{-2} + c$$

$$= -\frac{25}{(t^2+1)^2} + c$$

When $t = 2$

$$7 = -\frac{25}{(2^2+1)^2} + c$$

$$c = 8$$

$$\therefore v = -\frac{25}{(t^2+1)^2} + 8$$

Question 67

$$x = \int -10(2t-1)^{-2}dx$$
$$= -5\int 2(2t-1)^{-2}dx$$
$$= -5 \times \frac{(2t-1)^{-1}}{-1} + c$$
$$= \frac{5}{(2t-1)} + c$$

When $t = -1$

$$2 = \frac{5}{-1} + c$$
$$c = 7$$
$$\therefore x = \frac{5}{(2t-1)} + 7$$

Question 68

a

$$y = \int 24(2x-1)^3 dx$$
$$= 12\int 2(2x-1)^3 dx$$
$$= 12 \times \frac{(2x-1)^4}{4} + c$$
$$= 3(2x-1)^4 + c$$

When $x = 0$

$$5 = 3 + c$$
$$c = 2$$
$$\therefore y = 3(2x-1)^4 + 2$$

b $y = 3(2-1)^4 + 2$
$= 5$

c $245 = 3(2x-1)^4 + 2$
$243 = 3(2x-1)^4$
$81 = (2x-1)^4$

$(2x-1) = 3$ or $(2x-1) = -3$
$2x = 4$ $\quad$ $2x = -2$
$x = 2$ $\quad$ $x = -1$

Exercise 3C

Question 1

a $v = 6t^2 + 4$

$a = \dfrac{dv}{dt} = 12t$

At $t = 4$, $a = 48$ m/s^2

b $x = \int (6t^2 + 4)dt$

$= 2t^3 + 4t + c$

$5 = 2 + 4 + c$

$c = -1$

$\therefore x = 2t^3 + 4t - 1$

When $t = 2$,

$x = 2(2)^3 + 4(2) - 1$

$= 23$ m

Question 2

a $a = 6t - 2$

When $t = 1$

$a = 4$ m/s^2

b $v = \int (6t - 2)dt$

$= 3t^2 - 2t + c$

$1 = 3(0) - 2(0) + c$

$\therefore v = 3t^2 - 2t + 1$

When $t = 4$,

$v = 3(4)^2 - 2(4) + 1$

$= 9$ m/s

c $x = \int (3t^2 - 2t + 1)dt$

$x = t^3 - t^2 + t + c$

$5 = 0 - 0 + 0 + c$

$\therefore x = t^3 - t^2 + t + 5$

When $t = 3$,

$x = 27 - 9 + 3 + 5$

$= 26$ m

Question 3

a $a = 2t(5-6t)$

$v = \int a\,dt$

$= \int 2t(5-6t)\,dt$

$= \int (10t - 12t^2)\,dt$

$= 5t^2 - 4t^3 + c$

When $t = 0,\ v = 2$

$2 = 0 - 0 + c$

$c = 2$

$\therefore v = 5t^2 - 4t^3 + 2$

When $t = 2,$

$v = 5(4) - 4(8) + 2$

$= -10$ m/s

b 10 m/s

c $x = \int v\,dt$

$= \int (5t^2 - 4t^3 + 2)\,dt$

$= \frac{5}{3}t^3 - t^4 + 2t + c$

When $t = 0,\ x = 0$

$\therefore c = 0$

$x = \frac{5}{3}t^3 - t^4 + 2t$

When $t = 3,$

$x = \frac{5}{3}(3)^3 - (3)^4 + 2(3)$

$= \frac{5}{3} \times 27 - 81 + 6$

$= -30$ m

Question 4

$a = 6(t+1)^{-3}$

$t = 0,\ v = 2,\ x = 5$

$\therefore v = \int 6(t+1)^{-3} dt$

$$= \frac{6(t+1)^{-2}}{-2} + c$$

$$= -\frac{3}{(t+1)^2} + c$$

$$2 = -\frac{3}{1} + c$$

$$c = 5$$

$$v = \frac{-3}{(t+1)^2} + 5$$

When $t = 4$

$$v = \frac{-3}{25} + 5$$

$$= 4.88 \text{ m/s}$$

$$x = \int (-3(t+1)^{-2} + 5) dt$$

$$= \frac{-3(t+1)^{-1}}{-1} + 5t + c$$

$$t = \frac{3}{1} + 0 + c$$

$$c = 2$$

$$\therefore x = \frac{3}{(t+1)} + 5t + 2$$

When $t = 4$,

$$x = \frac{3}{5} + 20 + 2$$

$$= 22.6 \text{ m}$$

Question 5

$v = (t+1)^{-2}$

$x = \int (t+1)^{-2}\, dt$

$= \dfrac{(t+1)^{-1}}{-1} + c$

$= \dfrac{-1}{(t+1)} + c$

$3 = \dfrac{-1}{1} + c$

$c = 4$

$\Rightarrow x = -\dfrac{1}{(t+1)} + 4$

When $t = 4$,

$x = -\dfrac{1}{5} + 4$

$= 3.8\text{ m}$

Question 6

a $a = 2 + t^{\frac{1}{2}}$

$v = \int (2 + t^{\frac{1}{2}})dt$

$= 2t + \frac{2}{3}t^{\frac{3}{2}} + c$

At $t = 0$, $v = 0$, $c = 0$

$v = 2t + \frac{2}{3}t^{\frac{3}{2}}$

$t = 9,$

$v = 18 + \frac{2}{3}(3^2)^{\frac{3}{2}}$

$= 36$ m/s

b $x = \int (2t + \frac{2}{3}t^{\frac{3}{2}})dt$

$= t^2 + \frac{2}{3} \times \frac{2}{5}t^{\frac{5}{2}} + c$

$= t^2 + \frac{4}{15}t^{\frac{5}{2}} + c$

Initially at 0, $c = 0$.

$x = t^2 + \frac{4}{15}t^{\frac{5}{2}}$

$= 81 + \frac{4}{15} \times (3^2)^{\frac{5}{2}}$

$= 145.8$ m

Question 7

$x = 5t + 4t^{-1},\ t > 0$

$v = \dfrac{dx}{dt}$

$4 = 5 - \dfrac{4}{t^2}$

By ClassPad

$t = \pm 2$ but $t > 0$.

$\therefore t = 2.$

$x = 5(2) + \dfrac{4}{(2)^2}$

$= 12 \text{ m}$

Question 8

If $a = 8 - t,\ t \geq 0$

At $t = 0,\ x = 16,\ v = 20$

$v = \int (8 - t)dt$

$= 8t - \dfrac{t^2}{2} + c$

$20 = 0 - 0^2 + c$

$c = 20$

$v = 8t - \dfrac{t^2}{2} + 20$

$2 = 8t - \dfrac{t^2}{2} + 20$

By ClassPad : $t = -2,\ 18$

$\therefore t = 18$

$x = \int (8t - \dfrac{t^2}{2} + 20)dt$

$= 4t^2 - \dfrac{t^3}{6} + 20t + c$

$t = 0,\ x = 16,\ c = 16$

$x = 4t^2 - \dfrac{t^3}{6} + 20t + 16$

When $t = 18,$

$x = 4(18)^2 - \dfrac{18^3}{6} + 20(18) + 16$

$= 700 \text{ m}$

Question 9

$$a = \frac{48}{5}(2t+1)^2$$

$$v = \int \frac{48}{5}(2t+1)^2\,dt$$

$$= \frac{48}{5} \times \frac{(2t+1)^3}{6} + c$$

$$44 = 1.6(3)^3 + c$$

$$c = 0.8$$

$$\therefore v = 1.6(2t+1)^3 + 0.8$$

$$x = \int v dt = \frac{1.6(2t+1)^4}{4.2} + 0.8t + c$$

$$= \frac{(2t+1)^4}{5} + 0.8t + c$$

When $t = 1,\ x = 19$

$$19 = \frac{3^4}{5} + 0.8 + c$$

$$c = 2$$

$$\therefore x = \frac{(2t+1)^4}{5} + 0.8t + 2$$

Question 10

$a = 3t - 11$

$v = \int (3t - 11)dt$

$= \frac{3t^2}{2} - 11t + c$

When $t = 0,\ v = 14$

$\therefore c = 14$

$v = \frac{3t^2}{2} - 11t + 14$

$x = \int \left(\frac{3t^2}{2} - 11t + 14 \right) dt$

$= \frac{t^3}{2} - \frac{11t^2}{2} + 14t + c$

$t = 0,\ x = 0$

$\therefore c = 0$

When body at 0,

$0 = \frac{t^3}{2} - \frac{11t^2}{2} + 14t$

By ClassPad, $t = 0\text{s},\ 4\text{s},\ 7\text{s}$

At $t = 4$,

$v = \frac{3}{2}(4)^2 - 11(4) + 14$

$= -6$ m/s

Question 11

a $t = 0,\ v = 0,\ x = 0$

$a = 18 - 6t$

$v = \int (18 - 6t)dt$

$= 18t - 3t^2 + c$

Given $v = 0$ when $t = 0,\ \therefore c = 0$

so $v = 18t - 3t^2$

When body is at rest, $v = 0$

$0 = t(18 - 3t)$

$\therefore t = 0$ or $t = 6$

Body is next at rest at $t = 6$

$x = \int (18t - 3t^2)dt$

$= 9t^2 - t^3 + c$

$t = 0,\ x = 0,\ c = 0$

$\therefore x = 9t^2 - t^3$

When $t = 6,\ x = 9.6^2 - 6^3$

$= 108$ m

b x when $t = 5$,

$x = 9 \times 5^2 - 5^3$

$= 100$ m

x when $t = 6$,

$x = 108$ m

x when $t = 7$,

$x = 9 \times 7^2 - 7^3$

$= 98$ m

Distance of $t = 5$ to $t = 6 = 8$ m

Distance of $t = 6$ to $t = 7 = 10$ m

$\therefore$ Total distance $= 18$ m

Question 12

a $t = 0,\ x = 0,\ v = 0$

$a = 0.25$

$v = 0.25t + c$

but $t = 0,\ v = 0 \therefore c = 0$

$v = 0.25t$

$x = \int 0.25t dt$

$= 0.125t^2 + c$

When $t = 0,\ x = 0 \Rightarrow c = 0$

so $x = \dfrac{t^2}{8}$

$\therefore x = \dfrac{120^2}{8} = 1800 \text{ m}$

$= 1.8 \text{ km}$

b $v = 0.25(120)$

$= 30 \text{ m/s}$

Question 13

$a = a$

$t = 0,\ v = u,\ s = 0$

$v = \int a\ dt$

$= at + c$

When $t = 0,\ v = u$

$u = c$

$\therefore v = at + u$

$s = \int (u + at) dt$

$= ut + \dfrac{1}{2}at^2 + c$

When $t = 0,\ s = 0 \therefore c = 0$

$s = ut + \dfrac{1}{2}at^2$

Question 14

$a = 6t + 1$

When $t = 2,\ s = 12.$

When $t = 3,\ s = 34.$

$v = 3t^2 + t + c$

$x = \int v\, dt$

$= t^3 + \frac{t^2}{2} + ct + k$

At $t = 2,$

$12 = 8 + 2 + 2c + k$

$2c + k = 2$

At $t = 3,$

$34 = 27 + 4\frac{1}{2} + 3c + k$

$3c + k = 2\frac{1}{2}$

$k = 2 - 2c = 2\frac{1}{2} - 3c$

$c = \frac{1}{2}$

$\therefore k = 1$

$\therefore s = t^3 + \frac{t^2}{2} + \frac{t}{2} + 1$

When $t = 4,$

$s = 64 + 8 + 2 + 1$

$= 75$ m

$v = 3t^2 + t + \frac{1}{2}$

$= 3 \times 16 + 4 + \frac{1}{2}$

$= 52.5$ m/s

Question 15

$a = (3t + 2)\ \text{m/s}^2$

$t = 0,\ v > 0$

$s_4 - s_3 = 30$

$v = \int (3t + 2)dt$

$= \dfrac{3t^2}{2} + 2t + c$

$\therefore c > 0$ (body has initial positive velocity)

$s = \int \left(\dfrac{3t^2}{2} + 2t + c \right) dt$

$= \dfrac{t^3}{2} + t^2 + ct + k$

$s_4 - s_3 = (32 + 16 + 4c + k) - (13\dfrac{1}{2} + 9 + 3c + k) = 30$

$25.5 + c = 30$

$c = 4.5$

$\therefore v = \dfrac{3}{2}t^2 + 2t + 4.5$

When $t = 5$,

$v = \dfrac{3}{2} \times 25 + 10 + 4.5$

$= 52\ \text{m/s}$

Miscellaneous exercise three

Question 1

$$y = (x+3)(x^2+1)$$
$$\frac{dy}{dx} = (x+3)(2x) + (x^2+1)(1)$$
$$= 2x^2 + 6x + x^2 + 1$$
$$= 3x^2 + 6x + 1$$

Question 2

$$y = (x-5)(x^2-7)$$
$$\frac{dy}{dx} = (x-5)(2x) + (x^2-7)(1)$$
$$= 2x^2 - 10x + x^2 - 7$$
$$= 3x^2 - 10x - 7$$

Question 3

$$y = (x+1)(x^2+x+1)$$
$$\frac{dy}{dx} = (x+1)(2x+1) + (x^2+x+1)(1)$$
$$= 2x^2 + 3x + 1 + x^2 + x + 1$$
$$= 3x^2 + 4x + 2$$

Question 4

$$y = (2x-3)(x^2+5)$$
$$\frac{dy}{dx} = (2x-3)(2x) + (x^2+5)(2)$$
$$= 4x^2 - 6x + 2x^2 + 10$$
$$= 6x^2 - 6x + 10$$

Question 5

$$y = (3x-2)(3x^2+x-1)$$
$$\frac{dy}{dx} = (3x-2)(6x+1) + (3x^2+x-1)(3)$$
$$= 18x^2 - 9x - 2 + 9x^2 + 3x - 3$$
$$= 27x^2 - 6x - 5$$

Question 6

$$y = (4x+1)(x^2-5x+1)$$

$$\frac{dy}{dx} = (4x+1)(2x-5)+(x^2-5x+1)(4)$$

$$= 8x^2-18x-5+4x^2-20x+4$$

$$= 12x^2-38x-1$$

Question 7

a $f(x) = (2x-3)^5$

$$f'(x) = 5(2x-3)^4 \times 2$$

$$= 10(2x-3)^4$$

b $f'(2) = 10(2(2)-3)^4$

$$= 10$$

c $f''(x) = 4 \times 10(2x-3)^3 \times 2$

$$= 80(2x-3)^3$$

d $f''(2) = 80(2(2)-3)^3$

$$= 80$$

Question 8

$$y' = 3ax^2+2x+b$$

$$y'(2) = 3a(2)^2+2(2)+b = 50$$

$$12a+4+b = 50$$

$$12a+b = 46$$

$$y'' = 6ax+2 = 23$$

$$y''(1) = 6a(1)+2 = 23$$

$$6a = 21$$

$$a = 3.5$$

$$12(3.5)+b = 46$$

$$42+b = 46$$

$$b = 4$$

Question 9

$x = 2t^3 - 9t^2 + 5$

$v = \dfrac{dx}{dt} = 6t^2 - 18t$

$a = \dfrac{dv}{dt} = 12t - 18$

$12t - 18 = 0$

$12t = 18$

$t = 1.5$

When $t = 1.5$,

$v = 6(1.5)^2 - 18(1.5)$

$= -13.5$ m/s

When $t = 1.5$,

$x = 2(1.5)^3 - 9(1.5)^2 + 5$

$= -8.5$ m

Question 10

a $\dfrac{dy}{dx} = 6x + 2 = -10$

$6x = -12$

$x = -2$

When $x = -2$,

$y = 3(-2)^2 + 2(-2)$

$= 8$

$\therefore (-2, 8)$

b $\frac{dy}{dx} = 3x^2 - 5 = 43$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

When $x = 4$,

$y = 4^3 - 5(4)$

$= 44$

When $x = -4$,

$y = (-4)^3 - 5(-4)$

$= 44$

$\therefore (-4, -44)$ and $(4,\ 44)$

c $\frac{dy}{dx} = \frac{(3x+1)(-1) - 3(5-x)}{(3x+1)^2}$

$\frac{-16}{(3x+1)^2} = 1$

$(3x+1)^2 = 16$

$3x + 1 = \pm 4$

$3x = -1 \pm 4$

$x = \frac{-1 \pm 4}{3}$

$= -\frac{5}{3}, 1$

When $x = -\frac{5}{3}$,

$y = \frac{5 + \frac{5}{3}}{3 \times (\frac{5}{3}) + 1}$

$= -\frac{5}{3}$

When $x = 1$,

$y = \frac{5-1}{3+1}$

$= 1$

$\therefore (1,\ 1)$ and $\left(-\frac{5}{3}, -\frac{5}{3}\right)$

Question 11

a When $t = 0,\ x = 0,\ v = 0$

$a = 28 - 0.2t \qquad 0 \le t \le 120$

$v = \int (28 - 0.2t)dt$

$= 28t - \dfrac{0.2t^2}{2} + c$

$= 28t - 0.1t^2 + c$

At $t = 0,\ v = 0 \therefore c = 0$

$\therefore v = 28t - 0.1t^2$

$x = \int v dt$

$= \int (28t - 0.1t^2)dt$

$= \dfrac{28t^2}{2} - \dfrac{0.1t^3}{3} + c$

$= 14t^2 - \dfrac{t^2}{30} + c$

At $t = 0,\ x = 0 \therefore c = 0$

$\therefore x = 14t^2 - \dfrac{t^2}{30}$

After 1 minute, $t = 60$

$x = \left(14(60)^2 - \dfrac{60^3}{30}\right)\text{m}$

$= 43200 \text{ m}$

$= 43.2 \text{ km}$

b After 2 minute, $t = 120$

$x = \left(14(120)^2 - \dfrac{120^3}{30}\right)\text{m}$

$= 144 \text{ km}$

c After 3 minute, $t = 180$

$x = \left(14(180)^2 - \dfrac{180^3}{30}\right)\text{m}$

$= 259.2 \text{ km}$

Question 12

Marginal cost

$$\frac{dC}{dn} = \frac{1}{4}n^2 - 24n + 800$$

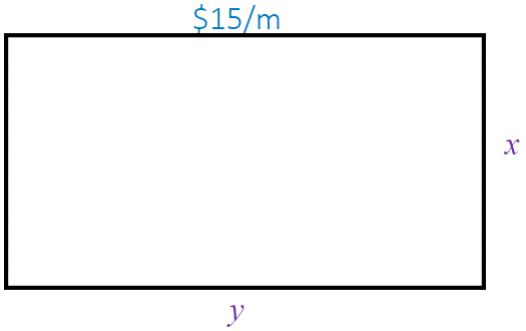
When $n = 100$,

$$\frac{dC}{dn} = \frac{1}{4} \times 100^2 - 24(100) + 800$$

$$= \$900$$

It means that the cost per item at this time is \$900 and the 101st item will cost \$900.

Question 13



$$xy = 300 \Rightarrow y = \frac{300}{x}$$

$$\text{Cost} = 9(2x + y) + 15y$$

$$= 18x + 9y + 15y$$

$$= 18x + 24y$$

$$= 18x + 24 \times \frac{300}{x}$$

$$= 18x + \frac{7200}{x}$$

$$\frac{dC}{dx} = 18 - \frac{7200}{x^2}$$

$$\frac{7200}{x^2} = 18$$

$$x^2 = \frac{7200}{18}$$

$$x^2 = 400$$

$$x = 20$$

$$y = \frac{300}{20}$$

$$= 15$$

$\therefore$ Measurements of area 15 m by 20 m with one of the 15 m sides having the \$15/m fencing.

Question 14

a **i** $C = \dfrac{1000p}{100 - p}$

$\dfrac{1000 \times 1}{100 - 1} = 10.10$

$\therefore \$10$

ii $C = \dfrac{1000 \times 5}{100 - 5}$

$= 52.63$

$\therefore \$53$

iii $C = \dfrac{1000 \times 50}{100 - 50}$

$= 1000$

$\therefore \$1000$

iv $C = \dfrac{1000 \times 95}{100 - 95}$

$= 19\ 000$

$\therefore \$19\ 000$

b $\dfrac{dC}{dp} = \dfrac{(100 - p)1000 - 1000p(-1)}{(100 - p)^2}$

$= \dfrac{100\ 000 - 1000p + 1000p}{(100 - p)^2}$

$= \dfrac{100\ 000}{(100 - p)^2}$

Question 15

a **i** $C = 5(16) + 120\sqrt{16} + 5000\ f'(x) =$
$= \$5560$

ii $C = 5(400) - 120\sqrt{400} + 5000$
$= \$9400$

b **i** $\frac{5560}{16} = \$347.50$

ii $\frac{9400}{400} = \$23.50$

c $\frac{dC}{dx} = 5 + \frac{60}{\sqrt{x}}$

d **i** At $x = 16$,
$\frac{dC}{dx} = 5 + \frac{60}{\sqrt{16}}$
$= \$20$

ii At $x = 400$,
$\frac{dC}{dx} = 5 + \frac{60}{\sqrt{400}}$
$= \$8$

e $C(17) - C(16)$
$= 5579.77 - 5560$
$= 19.77$
\$19.77 is close to \$20.

$C(401) - C(400)$
$= 9408.00 - 9400$
$= \$8$
This gives the same result.

Question 16

$$\frac{dy}{dx} = 3(2x+5)^2 \times 2 - \frac{54}{x^2}$$

$$0 = 6(2x+5)^2 - \frac{54}{x^2}$$

By ClassPad, $x = -3, -1.5, -1, 0.5$

When $x = -3$, $y = -19$ $(-3, -19)$

When $x = -1.5$, $y = -28$ $(-1.5, -28)$

When $x = -1$, $y = -27$ $(-1, -27)$

When $x = 0.5$, $y = 324$ $(0.5,\ 324)$

$\therefore$ There are four stationary points.

$$\frac{d^2y}{dx^2} = 6 \times 2(2x+5) \times 2 + \frac{108}{x^3}$$

$$= 24(2x+5) + \frac{108}{x^3}$$

When $x = -3$,

$$\frac{d^2y}{dx^2} = 24(-6+5) + \frac{108}{-27}$$

$$= -28$$

$\therefore (-3, -19)$ is a maximum point.

When $x = -1.5$,

$$\frac{d^2y}{dx^2} = 24(-3+5) + \frac{108}{(-1.5)^3}$$

$$= 16$$

$\therefore (-1.5, -28)$ is a minimum point.

When $x = -1$,

$$\frac{d^2y}{dx^2} = 24(-2+5) + \frac{108}{-1}$$

$$= -36$$

$\therefore (-1, -27)$ is a maximum point.

When $x = 0.5$,

$$\frac{d^2y}{dx^2} = 24(1+5) + \frac{108}{0.53}$$

$$= 1008$$

$\therefore (0.5,\ 324)$ is a minimum point.